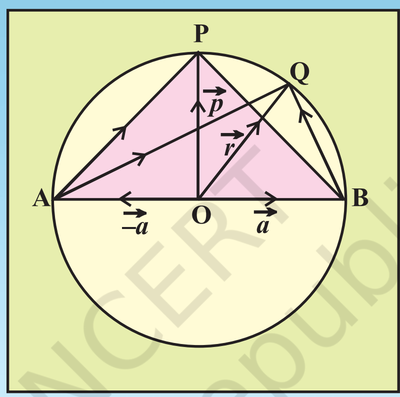
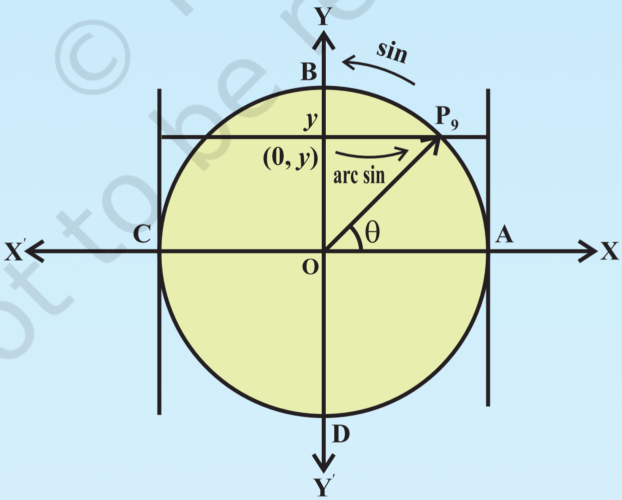
Activities for
Class XII





The basic principles of learning mathematics are : (a) learning should be related to each child individually (b) the need for mathematics should develop from an intimate acquaintance with the environment (c) the child should be active and interested, (d) concrete material and wide variety of illustrations are needed to aid the learning process (e) understanding should be encouraged at each stage of acquiring a particular skill (f) content should be broadly based with adequate appreciation of the links between the various branches of mathematics, (g) correct mathematical usage should be encouraged at all stages.

– Ronwill

Activity 1

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

Material Required

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

Method of Construction

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.

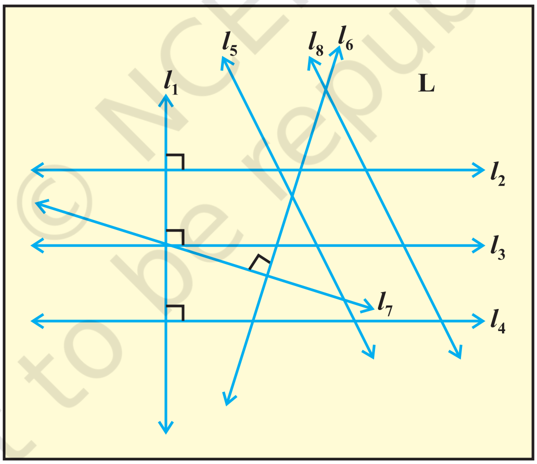


Fig. 1

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in$ R

Observation

1. In Fig. 1, no line is perpendicular to itself, so the relation R = $\{(l, m) : l \perp m\}$ ______ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? ______ (Yes/No)

   $\therefore$ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ______ R ($\notin / \in$)

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? ______ (Yes/No)

   $\therefore$ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ______ R ($\notin / \in$)

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? ______ (Yes/No)

   $\therefore$ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ ______ R ($\notin / \in$)

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in$ R and $(l_1, l_3) \in R \Rightarrow (l_2, l_3)$ ______ R ($\notin / \in$)

   $\therefore$ The relation R .... transitive (is/is not).

Application

This activity can be used to check whether a given relation is an equivalence relation or not.

Note

1. In this case, the relation is not an equivalence relation.
2. The activity can be repeated by taking some more wire in different positions.

Activity 2

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of plywood, some pieces of wire (8), plywood, nails, white paper, glue.

Method of Construction

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.

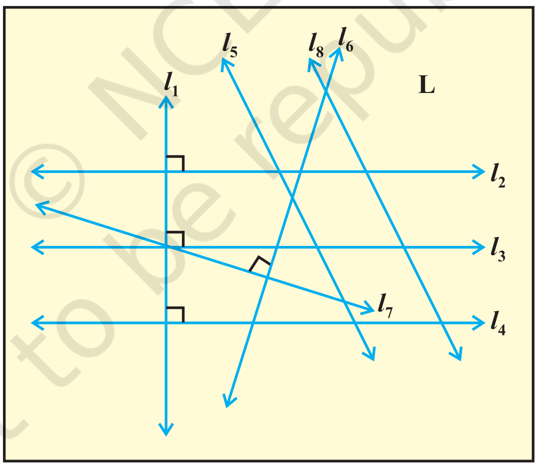


Fig. 2

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. (l_2, l_3), (l_3, l_4), (l_5, l_8), $\in$ R

OBSERVATION

1. In Fig. 2, every line is parallel to itself. So the relation R = $\{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)

2. In Fig. 2, observe that $l_2 \parallel l_3$. Is l_3 ... l_2? ($\nparallel / \parallel$)

So,	$(l_2, l_3) \in \text{R} \Rightarrow (l_3, l_2) \ldots \text{R}$ $(\notin / \in)$
Similarly,	$l_3 \parallel l_4$. Is $l_4 \ldots l_3$? $(\nparallel / \parallel)$
So,	$(l_3, l_4) \in \text{R} \Rightarrow (l_4, l_3) \ldots \text{R}$ $(\notin / \in)$
and	$(l_5, l_8) \in \text{R} \Rightarrow (l_8, l_5) \ldots \text{R}$ $(\notin / \in)$

$\therefore$ The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is l_2 ... l_4 ? $(\parallel / \parallel)$

So,	$(l_2, l_3) \in \text{R}$ and $(l_3, l_4) \in \text{R} \Rightarrow (l_2, l_4) \ldots \text{R}$ $(\in / \notin)$
Similarly,	$l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is $l_3 \ldots l_2$? $(\nparallel / \parallel)$
So,	$(l_3, l_4) \in \text{R}$, $(l_4, l_2) \in \text{R} \Rightarrow (l_3, l_2) \ldots \text{R}$ $(\in, \notin)$

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

APPLICATION

This activity is useful in understanding the concept of an equivalence relation.

NOTE

This activity can be repeated by taking some more wires in different positions.

Activity 3

OBJECTIVE

To demonstrate a function which is not one-one but is onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the Fig.3.1. Name the nails on the strip as 1, 2 and 3.
2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Fig.3.2. Name the nails on the strip as *a* and *b*.
3. Join nails on the left strip to the nails on the right strip as shown in Fig. 3.3.

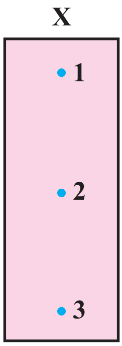


Fig. 3.1

Fig. 3.2

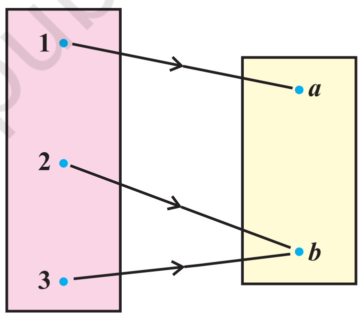


Fig. 3.3

DEMONSTRATION

1. Take the set X = {1, 2, 3}
2. Take the set Y = {*a*, *b*}
3. Join (correspondence) elements of X to the elements of Y as shown in Fig. 3.3

OBSERVATION

1. The image of the element 1 of X in Y is ___________.

   The image of the element 2 of X in Y is ___________.

The image of the element 3 of X in Y is __________.

So, Fig. 3.3 represents a __________ .

2. Every element in X has a _________ image in Y. So, the function is _________(one-one/not one-one).

3. The pre-image of each element of Y in X _________ (exists/does not exist). So, the function is ________ (onto/not onto).

APPLICATION

This activity can be used to demonstrate the concept of one-one and onto function.

NOTE

Demonstrate the same activity by changing the number of the elements of the sets X and Y.

Activity 4

Objective

To demonstrate a function which is one-one but not onto.

Material Required

Cardboard, nails, strings, adhesive and plastic strips.

Method of Construction

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails in it as shown in the Fig. 4.1. Name the nails as *a* and *b*.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the Fig. 4.2. Name the nails on the right strip as 1, 2 and 3.
3. Join nails on the left strip to the nails on the right strip as shown in the Fig. 4.3.

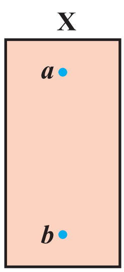


Fig. 4.1

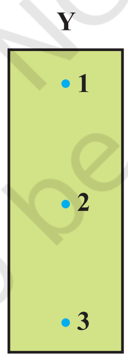


Fig. 4.2

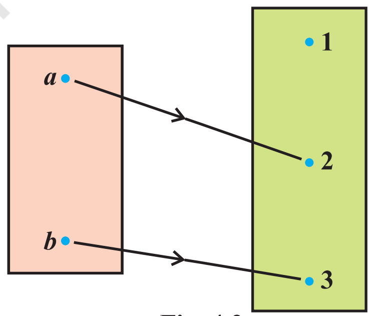


Fig. 4.3

Demonstration

1. Take the set X = $\{a, b\}$
2. Take the set Y = $\{1, 2, 3\}$.
3. Join elements of X to the elements of Y as shown in Fig. 4.3.

OBSERVATION

1. The image of the element *a* of X in Y is ______________.

   The image of the element *b* of X in Y is ______________.

   So, the Fig. 4.3 represents a _____________________.

2. Every element in X has a _________ image in Y. So, the function is _____________ (one-one/not one-one).

3. The pre-image of the element 1 of Y in X __________ (exists/does not exist). So, the function is __________ (onto/not onto).

   Thus, Fig. 4.3 represents a function which is _________ but not onto.

APPLICATION

This activity can be used to demonstrate the concept of one-one but not onto function.

Activity 5

Objective

To draw the graph of $\sin^{-1} x$, using the graph of sin x and demonstrate the concept of mirror reflection (about the line $y = x$).

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, pencil, eraser, cutter, nails and thin wires.

Method of Construction

1. Take a cardboard of suitable dimensions, say, 30 cm × 30 cm.
2. On the cardboard, paste a white chart paper of size 25 cm × 25 cm (say).
3. On the paper, draw two lines, perpendicular to each other and name them X′OX and YOY′ as rectangular axes [see Fig. 5].

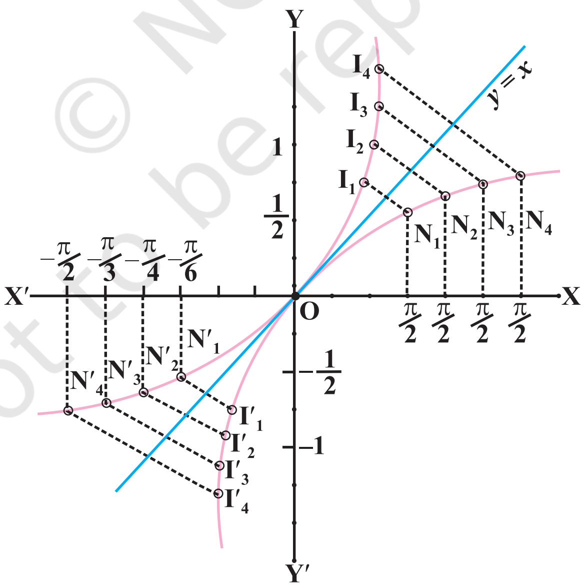


Fig. 5

4. Graduate the axes approximately as shown in Fig. 5.1 by taking unit on X-axis = 1.25 times the unit of Y-axis.

5. Mark approximately the points

   $\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ in the coordinate plane and at each point fix a nail.

6. Repeat the above process on the other side of the *x*-axis, marking the points $\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. Also fix a nail at O.

7. Join the nails with the help of a tight wire on both sides of *x*-axis to get the graph of sin *x* from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

8. Draw the graph of the line $y = x$ (by plotting the points (1,1), (2, 2), (3, 3), ... etc. and fixing a wire on these points).

9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. At these points fix nails, I_1, I_2, I_3, I_4.

10. Repeat the above activity on the other side of X- axis and fix nails at I_1', I_2', I_3', I_4'.

11. Join the nails on both sides of the line $y = x$ by a tight wire that will show the graph of $y = \sin^{-1} x$.

Demonstration

Put a mirror on the line $y = x$. The image of the graph of sin *x* in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of sin *x* and vice versa.

OBSERVATION

The image of point N_1 in the mirror (the line $y = x$) is _________.

The image of point N_2 in the mirror (the line $y = x$) is _________.

The image of point N_3 in the mirror (the line $y = x$) is _________.

The image of point N_4 in the mirror (the line $y = x$) is _________.

The image of point N_1' in the mirror (the line $y = x$) is _________.

The image point of N_2' in the mirror (the line $y = x$) is _________.

The image point of N_3' in the mirror (the line $y = x$) is _________.

The image point of N_4' in the mirror (the line $y = x$) is _________.

The image of the graph of six x in $y = x$ is the graph of _________, and the image of the graph of $\sin^{-1}x$ in $y = x$ is the graph of __________.

APPLICATION

Similar activity can be performed for drawing the graphs of $\cos^{-1}x$, $\tan^{-1}x$, etc.

Activity 6

OBJECTIVE

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

MATERIAL REQUIRED

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on it.
3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.
4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.
5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.

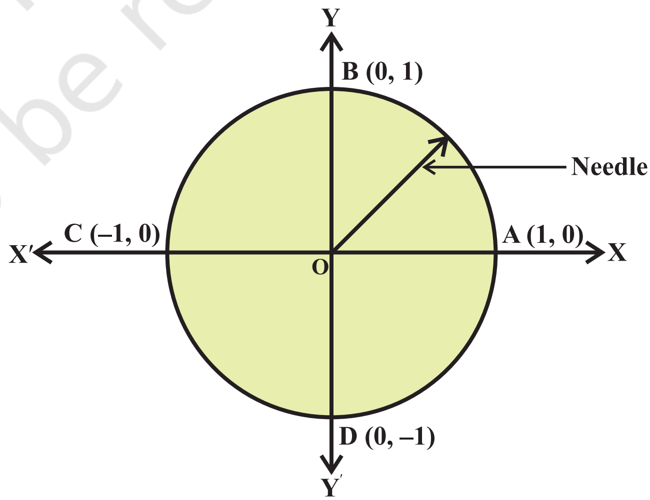


Fig. 6.1

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.

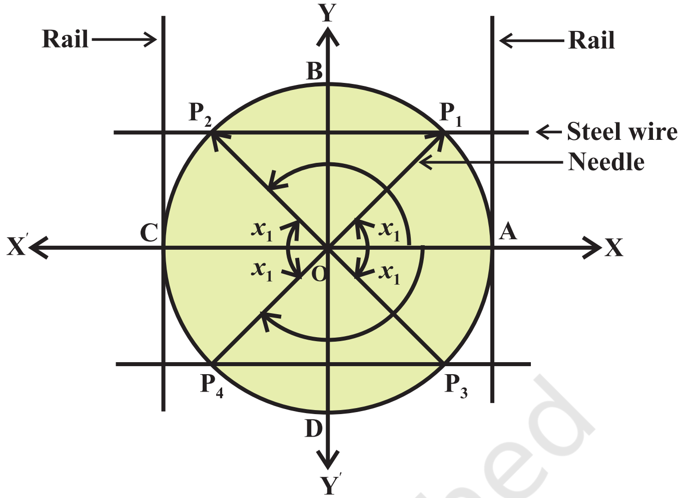


Fig. 6.2

DEMONSTRATION

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.

2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say P_1) (Fig. 6.2).

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$.

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate of intersecting point P_2 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin (\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly. y-co-ordinate of points P_5, P_6, P_7 and P_8 are different (see Fig. 6.3). Hence, sine function is one-to-one in

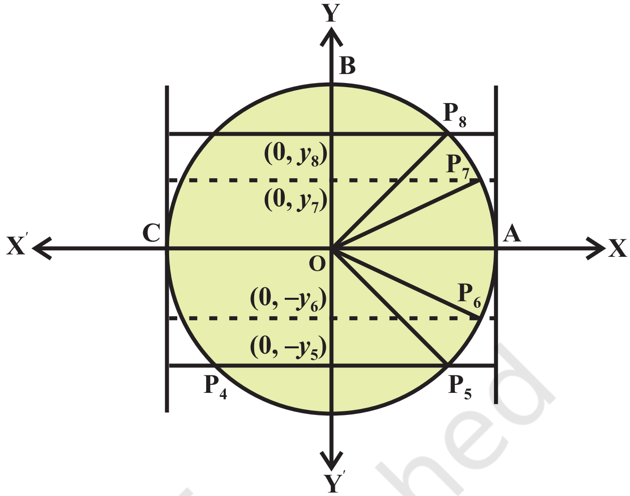


Fig. 6.3

the domian $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies between – 1 and 1.

7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as y. (see Fig. 6.4). Then $y = \sin\theta$ or $\theta = \text{arc} \sin^{-1} y$) as sine function is one-one and onto in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and range [–1, 1]. So, its inverse arc sine function exist. The domain of arc sine function is [–1, 1] and

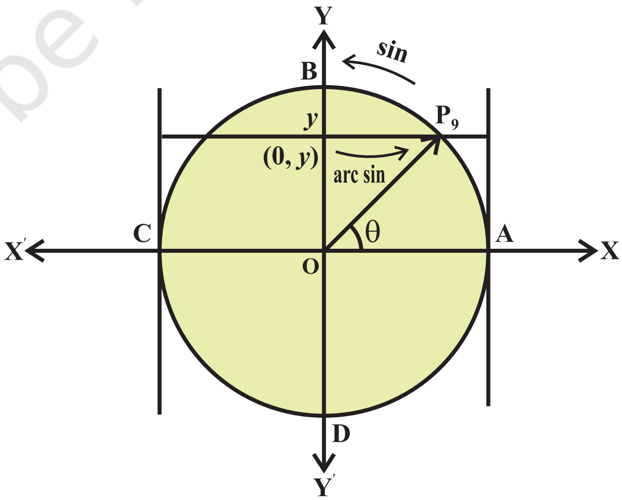


Fig. 6.4

range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

Observation

1. sine function is non-negative in _________ and __________ quadrants.
2. For the quadrants 3rd and 4th, sine function is _________.
3. θ = arc sin $y \Rightarrow y$ = ________ θ where $-\frac{\pi}{2} \leq \theta \leq$ ________.
4. The other domains of sine function on which it is one-one and onto provides _________ for arc sine function.

Application

This activity can be used for finding the principal value of arc cosine function ($\cos^{-1}y$).

Activity 7

Objective

To sketch the graphs of a^x and $\log_a x$, $a > 0$, $a \neq 1$ and to examine that they are mirror images of each other.

Material Required

Drawing board, geometrical instruments, drawing pins, thin wires, sketch pens, thick white paper, adhesive, pencil, eraser, a plane mirror, squared paper.

Method of Construction

1. On the drawing board, fix a thick paper sheet of convenient size 20 cm × 20 cm (say) with adhesive.

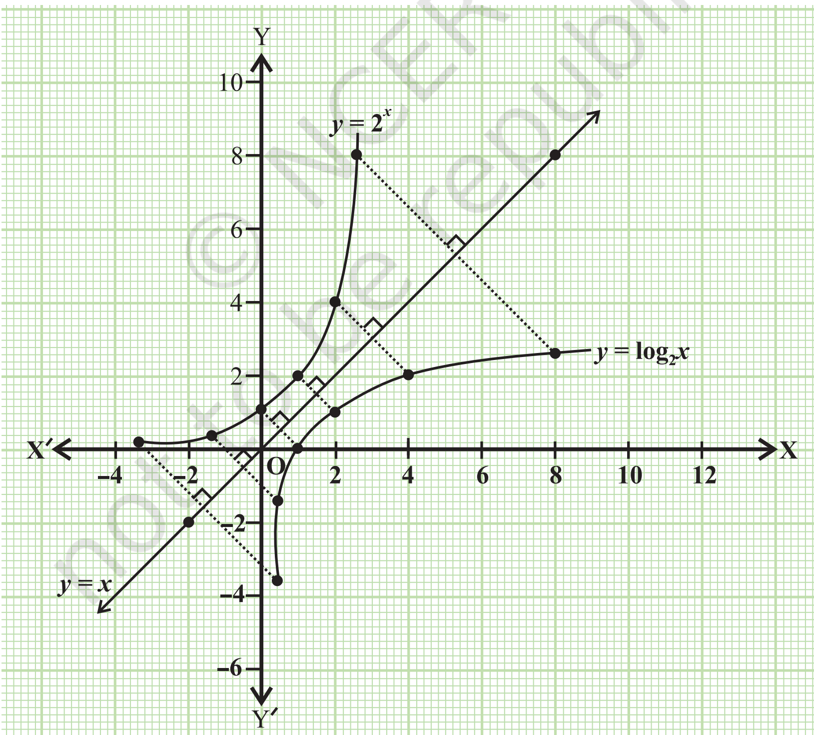


Fig. 7

2. On the sheet, take two perpendicular lines XOX′ and YOY′, depicting coordinate axes.

3. Mark graduations on the two axes as shown in the Fig. 7.

4. Find some ordered pairs satisfying $y = a^x$ and $y = \log_a x$. Plot these points corresponding to the ordered pairs and join them by free hand curves in both the cases. Fix thin wires along these curves using drawing pins.

5. Draw the graph of $y = x$, and fix a wire along the graph, using drawing pins.

Demonstration

1. For a^x, take $a = 2$ (say), and find ordered pairs satisfying it as

x	0	1	–1	2	–2	3	–3	$\frac{1}{2}$	$-\frac{1}{2}$	4
2^x	1	2	0.5	4	$\frac{1}{4}$	8	$\frac{1}{8}$	1.4	0.7	16

and plot these ordered pairs on the squared paper and fix a drawing pin at each point.

2. Join the bases of drawing pins with a thin wire. This will represent the graph of 2^x.

3. $\log_2 x = y$ gives $x = 2^y$. Some ordered pairs satisfying it are:

x	1	2	$\frac{1}{2}$	4	$\frac{1}{4}$	8	$\frac{1}{8}$
y	0	1	–1	2	–2	3	–3

Plot these ordered pairs on the squared paper (graph paper) and fix a drawing pin at each plotted point. Join the bases of the drawing pins with a thin wire. This will represent the graph of $\log_2 x$.

4. Draw the graph of line $y = x$ on the sheet.
5. Place a mirror along the wire representing $y = x$. It can be seen that the two graphs of the given functions are mirror images of each other in the line $y = x$.

Observation

1. Image of ordered pair (1, 2) on the graph of $y = 2^x$ in $y = x$ is ______. It lies on the graph of $y =$ _______.
2. Image of the point (4, 2) on the graph $y = \log_2 x$ in $y = x$ is _________ which lies on the graph of $y =$ _______.

Repeat this process for some more points lying on the two graphs.

Application

This activity is useful in understanding the concept of (exponential and logarithmic functions) which are mirror images of each other in $y = x$.

Activity 8

OBJECTIVE

To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base *e*) of the number *x*.

MATERIAL REQUIRED

Hardboard, white sheet, graph paper, pencil, scale, log tables or calculator (graphic/scientific).

METHOD OF CONSTRUCTION

1. Paste a graph paper on a white sheet and fix the sheet on the hardboard.
2. Find some ordered pairs satisfying the function $y = \log_{10} x$. Using log tables/ calculator and draw the graph of the function on the graph paper (see Fig. 8)

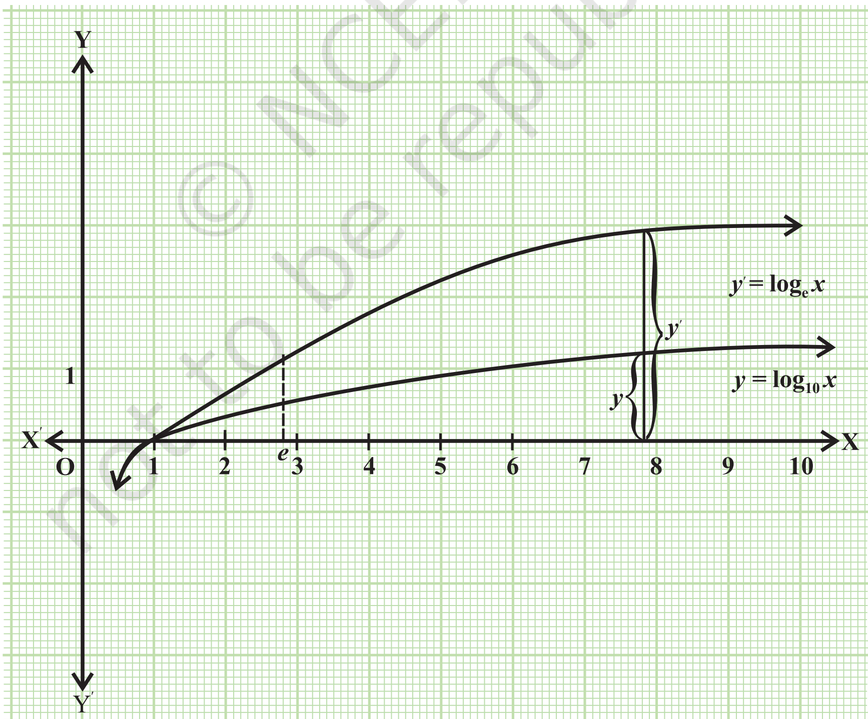


Fig. 8

3. Similarly, draw the graph of $y' = \log_e x$ on the same graph paper as shown in the figure (using log table/calculator).

DEMONSTRATION

1. Take any point on the positive direction of x-axis, and note its x-coordinate.
2. For this value of x, find the value of y-coordinates for both the graphs of $y = \log_{10} x$ and $y' = \log_e x$ by actual measurement, using a scale, and record them as y and y', respectively.
3. Find the ratio $\frac{y}{y'}$.
4. Repeat the above steps for some more points on the x-axis (with different values) and find the corresponding ratios of the ordinates as in Step 3.
5. Each of these ratios will nearly be the same and equal to 0.4, which is approximately equal to $\frac{1}{\log_e 10}$.

OBSERVATION

S.No.	Points on the x-axis	$y = \log_{10} x$	$y' = \log_e x$	Ratio $\frac{y}{y'}$ (approximate)
1.	x_1= ______	y_1 = ______	y'_1 = ______	___________
2.	x_2=______	y_2 = ______	y'_2 = ______	___________
3.	x_3=______	y_3 = ______	y'_3 = ______	___________
4.	x_4=______	y_4 = ______	y'_4 = ______	___________
5.	x_5=______	y_5 = ______	y'_5 = ______	___________
6.	x_6=______	y_6 = ______	y'_6 = ______	___________

2. The value of $\frac{y}{y'}$ for each point x is equal to __________ approximately.

3. The observed value of $\frac{y}{y'}$ in each case is approximately equal to the value of $\frac{1}{\log_e 10}$. (Yes/No)

4. Therefore, $\log_{10} x = \frac{\quad}{\log_e 10}$.

Application

This activity is useful in converting log of a number in one given base to log of that number in another base.

Note

Let, $y = \log_{10} x$, i.e., $x = 10^y$.

Taking logarithm to base e on both the sides, we get $\log_e x = y \log_e 10$

or $y = \frac{1}{\log_e 10}(\log_e x)$

$$\Rightarrow \frac{\log_{10} x}{\log_e x} = \frac{1}{\log_e 10} = 0.434294 \text{ (using log tables/calculator).}$$

Activity 9

Objective

To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point.

Material Required

Paper, pencil, calculator.

Method of Construction

1. Consider the function given by $f(x) = \begin{cases} \dfrac{x^2-16}{x-4}, & x \neq 4 \\ 10, & x = 4 \end{cases}$

2. Take some points on the left and some points on the right side of c (= 4) which are very near to c.

3. Find the corresponding values of $f(x)$ for each of the points considered in step 2 above.

4. Record the values of points on the left and right side of c as x and the corresponding values of $f(x)$ in a form of a table.

Demonstration

1. The values of x and $f(x)$ are recorded as follows:

Table 1 : For points on the left of c (= 4).

x	3.9	3.99	3.999	3.9999	3.99999	3.999999	3.9999999
$f(x)$	7.9	7.99	7.999	7.9999	7.99999	7.999999	7.9999999

2. **Table 2**: For points on the right of c (= 4).

x	4.1	4.01	4.001	4.0001	4.00001	4.000001	4.0000001
$f(x)$	8.1	8.01	8.001	8.0001	8.00001	8.000001	8.0000001

Observation

1. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the left.
2. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the right.
3. So, $\lim_{x \to 4^-} f(x)$ = ________ and $\lim_{x \to 4^+} f(x) =$ ________.
4. Therefore, $\lim_{x \to 4} f(x) =$ ________ , $f(4)$ = ________.
5. Is $\lim_{x \to 4} f(x) = f(4)$ ________ ? (Yes/No)
6. Since $f(c) \neq \lim_{x \to c} f(x)$, so, the function is ________ at x = 4 (continuous/not continuous).

Application

This activity is useful in understanding the concept of limit and continuity of a function at a point.

Activity 10

Objective

To verify that for a function f to be continuous at given point x_0,

$\Delta y = |f(x_0 + \Delta x) - f(x_0)|$ is

arbitrarily small provided. Δx is sufficiently small.

Material Required

Hardboard, white sheets, pencil, scale, calculator, adhesive.

Method of Construction

1. Paste a white sheet on the hardboard.
2. Draw the curve of the given continuous function as represented in the Fig. 10.
3. Take any point A (x_0, 0) on the positive side of x-axis and corresponding to this point, mark the point P (x_0, y_0) on the curve.

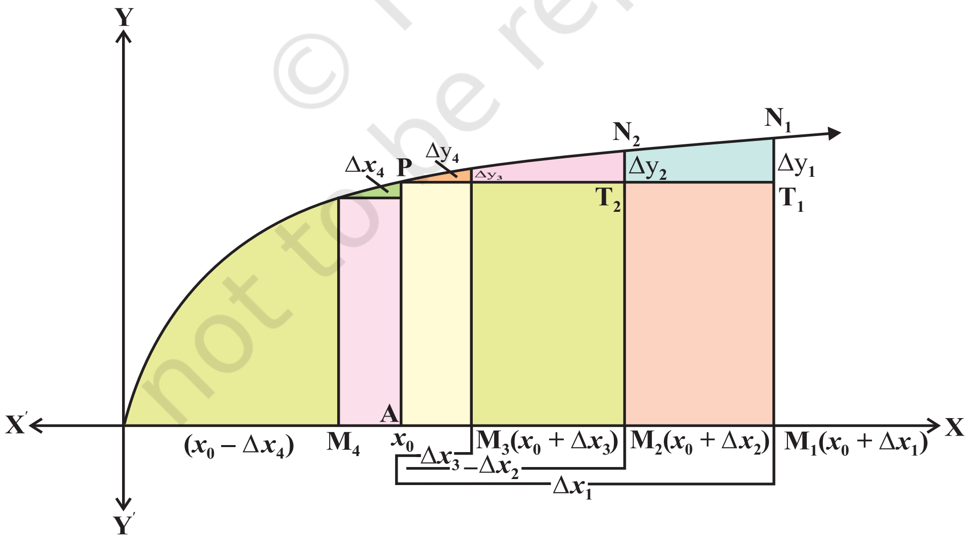


Fig. 10

Demonstration

1. Take one more point M_1 $(x_0 + \Delta x_1, 0)$ to the right of A, where Δx_1 is an increment in x.
2. Draw the perpendicular from M_1 to meet the curve at N_1. Let the coordinates of N_1 be $(x_0 + \Delta x_1, y_0 + \Delta y_1)$
3. Draw a perpendicular from the point P (x_0, y_0) to meet N_1M_1 at T_1.
4. Now measure $AM_1 = \Delta x_1$ (say) and record it and also measure $N_1T_1 = \Delta y_1$ and record it.
5. Reduce the increment in x to Δx_2 (i.e., $\Delta x_2 < \Delta x_1$) to get another point $M_2\left(x_0 + \Delta x_2, 0\right)$. Get the corresponding point N_2 on the curve
6. Let the perpendicular PT_1 intersects N_2M_2 at T_2.
7. Again measure $AM_2 = \Delta x_2$ and record it.

   Measure $N_2T_2 = \Delta y_2$ and record it.
8. Repeat the above steps for some more points so that Δx becomes smaller and smaller.

Observation

S.No.	Value of increment in x_0	Corresponding increment in y
1.	$\lvert\Delta x_1\rvert =$ ________	$\lvert\Delta y_1\rvert =$ ________
2.	$\lvert\Delta x_2\rvert =$ ________	$\lvert\Delta y_2\rvert =$ ________
3.	$\lvert\Delta x_3\rvert =$ ________	$\lvert\Delta y_3\rvert =$ ________
4.	$\lvert\Delta x_4\rvert =$ ________	$\lvert\Delta y_4\rvert =$ ________
5.	$\lvert\Delta x_5\rvert =$ ________	$\lvert\Delta y_5\rvert =$ ________

6.	$\lvert \Delta x_6 \rvert =$	$\lvert \Delta y_6 \rvert =$
7.	$\lvert \Delta x_7 \rvert =$	$\lvert \Delta y_7 \rvert =$
8.	$\lvert \Delta x_8 \rvert =$	$\lvert \Delta y_8 \rvert =$
9.	$\lvert \Delta x_9 \rvert =$	$\lvert \Delta y_9 \rvert =$
10.		

2. So, Δy becomes __________ when Δx becomes smaller.

3. Thus $\lim_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

APPLICATION

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

Projects

Project work in mathematics may be performed individually by a student or jointly by a group of students. These projects may be in the form of construction such as curve sketching or drawing of graphs, etc. It may offer a discussion of a topic from history of mathematics involving the historical development of particular subject in mathematics/ topics on concepts. Students may be allowed to select the topics of their own choice for projects in mathematics. The teacher may act as a facilitator by creating interest in various topics. Once the topic has been selected, the student should read as much about the topic as is available and finally prepare the project.

Project 1

To minimise the cost of the food, meeting the dietary requirements of the staple food of the adolescent students of your school.

Task to be done

(i) Make a survey of atleast 100 students to find which staple food they consume on daily basis.

(ii) Select two food items constituting one cereal and one pulse.

(iii) Find from dietician the minimum requirement of protein and carbohydrate for an adolescent and also find the content of protein and carbohydrate in 1 kg. of selected cereal and pulse respectively.

(iv) Find the minimum cost of the selected cereal and pulse from market.

(v) Formulate the corresponding Linear Programming problem.

(vi) Solve the problem graphically.

(vii) Interpret the result.

Project 2

Estimation of the population of a particular region/country under the assumptions that there is no migration in or out of the existing population in a particular year.

Task to be done

1. Find the population of a selected region in a particular year.

2. Find the number of births and number of deaths in the existing population in a particular year t (say). Let

   P(t): denote the population in a particular year t

   B(t): denote the number of births in one year between t and $t + 1$.

   D(t): denote the number of deaths in one year between t and $t + 1$.

3. Obtain the relation

   $$P(t + 1) = P(t) + B(t) - D(t) \qquad (1)$$

4. Assume that

   $b = \dfrac{B(t)}{P(t)}$ represents the birth rate for the time interval t to $t + 1$.

   $d = \dfrac{D(t)}{P(t)}$ represents death rate for the time interval t to $t + 1$.

5. From (1), we have

   $$P(t + 1) = P(t) + B(t) - D(t)$$

   $$= P(t)\left[1 + \frac{B(t)}{P(t)} - \frac{D(t)}{P(t)}\right]$$

   $$= P(t)(1 + b - d) \qquad (2)$$

6. Taking $t = 0$ in equation (2), we get

$P(1) = P(0)(1 + b - d)$.

For $t = 1$, we get

$P(2) = P(0)(1 + b - d)^2$.

Continuing above equation, we get

$$P(t) = P(0)(1 + b - d)^t \quad (3)$$

Here, it is assumed that birth rate and death rate remains the same for consecutive years. P(0) denote the initial population. Equation (3) gives the mathematical model for calculation the population in t year.

7. Using calculator find the population in different number of years.

8. Compare the population data obtained theoretically and draw the inferences.

Project 3

Finding the coordinates of different points identified in your classroom using the concepts of three dimensional geometry and also find the distances between the identified points.

Tasks to be done

1. Choose any corner of your classroom as the origin.
2. Take three perpendicular edges of walls as x–, y– and z-axes.
3. Find the coordinates of each corner of the room, corners of windows, doors and blackboard etc.
4. Find the coordinate of the tips of ceiling fan, bulbs and all other possible points in the space of the classroom.
5. Find the distances between different points by measurement as well as by using distance formula.
6. Find the coordinates of the diagonals of the room and length of the diagonals by distance formula.

Project 4

Formation of differential equation to explain the process of cooling of boiled water to a given room temperature.

Task to be done

1. Boil 1 litre of water in a pan/beaker.
2. Note the room temperature and the temperature of the boiled water.
3. Note the temperature at an interval of every half hour till the temperature of the water reaches the room temperature. Prepare a corresponding table as shown below:

Time (t) at an interval of $\frac{1}{2}$ hour	Temperature of water (T)	Room Temperature (P)	Difference T – P

4. Let T denote the temperature of the boiled water at time t. P denote the room temperature under the assumption it remains constant throughout the experiment.

$$\frac{dT}{dt} \propto T - P.$$

or $\frac{dT}{dt} = -k\,(T - P)$, k is proportionality constant and minus sign signifier that temperature is decreasing.

or $\frac{dT}{T - P} = -k dt$. Integrating, we have

$$\log |T - P| = -kt + C \qquad (1)$$

5. Find the value of C and k by using two initial values of T and t from the observation table to get the particular solution of the differential equation (1).

List of Projects

1. Project on history of Mathematicians: It may include history of Indian mathematicians such as Aryabhata, Brahmgupta, Varahamihir, Sridhara, Bhaskaracharya, Ramanujan etc., and history of foreign mathematicians such as Cantor, Pythagoras, Thales, Euclid, Appollonius, Descartes, Fermat, Leibnitz, Euler, Fibonac, Gauss, Newton, etc.
2. On linear Programming problems related to day-to-day life like collecting data from families of their expenditures and requirements from the factories to maximum out put.
3. Collect data from dieticians, transporters, agents and formulate linear programming problems.
4. Make a chart of the formulae of applications of calculus.
5. Applications of conic sections, vectors, three dimensional geometry, calculus, etc., in Mathematics and Physics.
6. Mathematics and Chemistry: Study structure of organic compounds.
7. Mathematics and Biology: Study of science of heredity etc.
8. Mathematics and Music
9. Mathematics and Environment
10. Mathematics and Arts: Construction of shapes using curves
11. Mathematics and Information and Communication Technology: Writing of Mathematical programmes, flow charts, algorithm, circuit diagrams etc.
12. Collection of statistical data and analysing it for standard deviation and mean deviation.
13. Observe the various patterns and properties in Pascal's triangle and make a project.

14. Prepare a project based on the Fibonacci sequence, their properties and similar pattern found in nature.

15. Form a differential equation for the growth of bacteria in different environments.

16. Study the nature of mathematics and make a project showing where three aspects of nature of mathematics - formalism, logic, intuition is applied in the development of mathematics.

Scheme of Evaluation

The following weightage are assigned for evaluation at Higher Secondary Stage in mathematics:

Theory Examination	:	80 marks
Internal Assessment	:	20 marks

1. Internal assessment of 20 marks, based on school based examination will have following break-up:

Year-end assessment of activities	:	12 marks
Assessment of Project Work	:	5 marks
Viva-voice	:	3 marks

- **Assessment of Activity Work**
  - (a) Every student will be asked to perform two given activities during the allotted time.
  - (b) The assessment may be carried out by a team of two mathematics teachers, including the teacher who is taking practical classes.
  - (c) The break-up of 12 marks for assessment for a single activity may be as under:

• Statement of objective of the activity	:	1 mark
• Material required	:	1 mark
• Preparation for the activity	:	3 marks
• Conduct of the activity	:	3 marks
• Observation and analysis	:	3 marks
• Results and Conclusion	:	1 mark
Total	**:**	**12 marks**

  - (d) The marks for two activities may be added first and then marks calculated out of 12.
  - (e) Full record of activities may be kept by each student.

- **Evaluation of Project Work**
  - (a) Every student will be asked to do at least one project based on the concepts learnt in the classroom.
  - (b) The project may be carried out individually (or in a group of two or three students).
  - (c) The weightage of 5 marks for the project may be as under :

• Identification and statement of the project	:	1 mark
• Planning the project	:	1 mark
• Procedure adopted	:	1 mark
• Observations from data collected	:	1 mark
• Interpretation and application of result	:	1 mark

Total Score out of 20 : The marks obtained in year-end assessment of activities and project work be added to the marks in viva-voice to get the total score out of 20.

Note : Every student should be asked to perform at least twenty activities in one academic year.

Set-up of Mathematics Laboratory