RELATIONS AND FUNCTIONS

1. Let **N** be the set of natural numbers and R be the relation in **N** defined as $R = \{(a, b) : a = b - 2, b > 6\}$. Then

   (A) $(2, 4) \in R$ (B) $(3, 8) \in R$

   (C) $(6, 8) \in R$ (D) $(8, 7) \in R$.

2. If A = { 1, 2, 3}, then the relation R = {(1, 2), (2, 3), (1, 3)} in A is ___.
   a. transitive only
   b. reflexive only
   c. symmetric only
   d. symmetric and transitive only
3. If n(A) = p and n(B) = q, then the number of relations from set A to set B = ________
4. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c on the set N×N is an equivalence relation.
5. Assertion (A): The function $f: R - \{(2n + 1)\, \pi/2: n \in Z\} \to (-\infty, -1] \cup [1, \infty)$ defined by f(x)= sec x is not one-one function in its domain.

   Reason (R): The line y = 2 meets the graph of the function at more than one point
6. If A= {1,2,3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)) is a function from A to B. State whether f is one-one or not.
7. If R= {(x, y): x + 2y = 8} is a relation on N, then write the range of R.
8. Show that the relation R in the set {1, 2, 3} given by R= {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)} is reflexive but neither symmetric nor transitive.
9. If A={1,2,3,...,9} and R be the relation in AxA defined by (a, b) R (c,d). If a +d=b+c for (a,b), (c,d) in Ax A. Prove that R is an equivalence relation, Also, obtain the equivalence class [(2, 5)]
10. If Z is the set of all integers and R is the relation on Z defined as R={(a,b): a,b∈Z and a - b is divisible by 5}. Prove that R is an equivalence relation.
11. If A = {2, 3, 4, 5} then write whether each of the following relations on set A is a function or not? Give reasons also.

    (i) {(2, 3), (3, 4), (4, 5), (5, 2)}

    (ii) {(2, 4), (3, 4), (5, 4), (4, 4)}

    (iii) {(2, 3), (2, 4), (5, 4)}

    (iv) {(2, 3), (3, 5), (4, 5)}

    (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

12. Check the following relations for each of (i) Reflexivity; (ii) Symmetricity; (iii) Transitivity; (iv) Equivalence Relation.

(a) R_1 = {(A, B); |A| = |B|, A, B are line segments in the same plane}

(b) R_2 = {(a, b), (b, b), (c, c), (a, c), (b, c)} in the set A = {a, b, c}

(c) R_3 = {(a, b) : a ≥ b, a, b, ∈ R}

(d) R_4 = {(a, b) : a divides b, a, b Î A } where A = {2, 3, 4, 5}

(e) R_5 = {a, b), (b, a), (a, a)} in {a, b, c}.

(f) R_6 = {(a, b) : a ≥ b, a, b ∈ N}

(g) R_7 = {(a, b) : a, b ∈ R, $a \le b^3$}

(h) R_8 = {(a, b) : a – b is multiple of 5, a, b, ∈ R}

(i) R_9 = {(a, b) : b = 3a and a, b ∈ R}

(j) R_{10} = {(a, b) : a – b is an integer, a, b ∈ R}

INVERSE TRIGONOMETRIC FUNCTIONS

13. Write the value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.

14. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

15. If $\sin(\sin^{-1}1/5 + \cos^{-1}x) = 1$, then find the value of x.

16. If $\tan^{-1}x + \tan^{-1}y = \pi/4$; $xy < 1$, then write the value of $x + y + xy$.

17. Write the value of $\cos^{-1}(-1/2) + 2\sin^{-1}(1/2)$.

18. Write the principal value of $\cos^{-1}[\cos(680)°]$.

19. Write the principal value of $\tan^{-1}[\sin(-\pi/2)]$

20. Find the value of the following. $\cot(\pi/2 - 2\cot^{-1}\sqrt{3})$

21. Write the principal value of $\tan^{-1}(1) + \cos^{-1}(-1/2)$.

22. Write the value of $\tan(2\tan^{-1}1/5)$.

23. Write the value of $\cos^{-1}(\cos 7\pi/6)$.

24. What is the principal value of $\cos^{-1}(\cos 2\pi/3) + \sin^{-1}(\sin 2\pi/3)$?

25. What is the principal value of $\tan^{-1}(-1)$?

26. Using the principal values, write the value of $\sin^{-1}(-\sqrt{3}/2)$.

27. Prove that

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, 0 < x < \frac{\pi}{2}$$

28. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1.$

29. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

30. If $\sin^{-1}x = y$, then

(A) $0 \le y \le \pi$ (B) $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$ (D) $\frac{-\pi}{2} < y < \frac{\pi}{2}$

Chapter – Three Dimensional Geometry Class – XII

1. Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=z$ & $x=\frac{y}{2}=\frac{z}{3}$ are perpendicular.
2. Find the equation of a line parallel to x axis and passes through origin.
3. Find the angle between lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$
4. Find the value of λ such that the lines $\frac{1-x}{3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ & $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{7}$ are perpendicular.
5. Find the equation of a line passes through the points P(−1, 3, 2) and Q(−4, 2, −2)). Also, if the point R(5, 5, λ) is collinear with the points P and Q then find the value of λ.
6. Find the equation of line passes through the point (2, −1, 3) and perpendicular to the lines $\bar{r}=(\hat{\imath}+\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}-2\hat{\jmath}+\hat{k})$ and $\bar{r}=(2\hat{\imath}-\hat{\jmath}-3\hat{k})+\mu(\hat{\imath}+2\hat{\jmath}+2\hat{k})$.
7. Find the point of intersection of the lines $\bar{r}=(3\hat{\imath}+2\hat{\jmath}-4\hat{k})+\lambda(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ and $\bar{r}=(5\hat{\imath}-2\hat{\jmath})+\mu(3\hat{\imath}+2\hat{\jmath}+6\hat{k})$
8. Find the relation between lines $\bar{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\bar{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$ and hence find the shortest distance between them.
9. Find the equation of straight lines passes through origin which intersects the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angle of $\frac{\pi}{3}$ each.
10. Find the value of a such that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-a}{4}$ and $\frac{x-4}{4}=\frac{y-1}{2}=z$ are skew lines.
11. Find the coordinates of foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\bar{r}=(-\hat{\imath}+3\hat{\jmath}+\hat{k})+\lambda(2\hat{\imath}+3\hat{\jmath}-\hat{k})$. Also find the image of the point with respect to the line.
12. A line makes angles α, β, γ, δ with the four diagonals of a cube. Prove that $cos^2\alpha+cos^2\beta+cos^2\gamma+cos^2\delta=\frac{4}{3}$
13. Find the value of p, so that lines $\frac{x-2}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{1-z}{7}$ are perpendicular to each other.
14. Find the shortest distance between the lines $\vec{r}=(4\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\vec{r}=(\hat{\imath}-\hat{\jmath}+2\hat{k})+\mu(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$
15. Find the angle between following pair of lines $\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$ and check whether the lines are parallel or perpendicular.
16. Find the image of the point (2, -1, 5) in the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.
17. If vectors $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\hat{k}$, $\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}$ are such that $\vec{b}+\lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.
18. The two co-initial adjacent sides of a parallelogram are $2\hat{\imath}-4\hat{\jmath}-5\hat{k}$ and $2\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram.
19. An ant is moving along the vector $l_1=\hat{\imath}-2\hat{\jmath}+3\hat{k}$. Few sugar crystals are kept along the vector $l_2=3\hat{\imath}-2\hat{\jmath}+\hat{k}$ which are inclined at an angle θ with the vector l_1. Then find the angle θ. Also, find the scalar projection of l_1 on l_2.
20. Find the vector and the cartesian equation of the line that passes through (-1, 2, 7) and is perpendicular to the lines $\vec{r}=(2\hat{\imath}+\hat{\jmath}-3\hat{k})+\lambda(\hat{\imath}+2\hat{\jmath}+5\hat{k})$ and $\vec{r}=(3\hat{\imath}+3\hat{\jmath}-7\hat{k})+\mu(3\hat{\imath}-2\hat{\jmath}+5\hat{k})$.

21. Find the shortest distance between the lines $\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$ and $\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$, where λ and μ are parametres.
22. Find the image of the point (1, 2, 1) with respect to the line $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3}$. Also find the equation of the line joining the given point and its image .
23. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$ and hence verify your answer.
24. Find the coordinates of the foot of the perpendicular drawn from the point (2, 3, -8) in the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the perpendicular distance of the given point from the line.
25. Find the shortest distance between the lines l_1 and l_2 given below
    l_1 : The line passing through (2, -1, 1) and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$ $l_2 : \vec{r} = \hat{\imath} + (2\mu + 1)\hat{\jmath} - (\mu + 2)\hat{k}$.
26. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1) . Also, find the angle between the given lines.
27. Find the shortest distance between the lines given by $\vec{r} = (2 + \lambda)\hat{\imath} - (3 + \lambda)\hat{\jmath} + (5 + \lambda)\hat{k}$ and $\vec{r} = (2\mu - 1)\hat{\imath} + (4\mu - 1)\hat{\jmath} + (5 - 3\mu)\hat{k}$).
28. Vertices B and C of ΔABC lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates (1, -1, 2) and line segment BC has length 5 units.
29. For any two vectors $\vec{a}$ and $\vec{b}$, show that $(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) = (1 - \vec{a}.\vec{b})^2 + |\vec{a} + \vec{b} + (\overrightarrow{a \times b})|^2$
30. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \hat{c} = \hat{b}$ and $\vec{a}.\vec{c} = 3$.
31. Find the value of p, so that the lines $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equation of a line passing through a point (3, 2, -4) and parallel to the line l_1.
32. Find the value of λ, so that the vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$ are perpendicular to each other.
33. Find the area of the triangle whose two sides are represented by the vectors $2\hat{\imath}$ and $-3\hat{\jmath}$.
34. Find the shortest distance between the lines $\frac{x-4}{4} = \frac{y+1}{2} = \frac{z}{1}$ and $\frac{x-1}{5} = \frac{y-2}{2} = \frac{z-3}{1}$.
35. Find the equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines $\vec{r} = (8\hat{\imath} - 19\hat{\jmath} + 10\widehat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ and $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$.
36. Find a vector of magnitude 5 units, perpendicular to each of the vectors ($\vec{a}$+ $\vec{b}$) and ($\vec{a}$- $\vec{b}$), where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.
37. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, then find a unit vector perpendicular to both of the vectors ($\vec{a}$- $\vec{b}$) and ($\vec{c}$- $\vec{b}$) .
38. Find the position vector of a point C which divides the line segment joining A and B whose position vectors are $2\vec{a}$+ $\vec{b}$ and $\vec{a}$- $3\vec{b}$, externally in the ratio 1:2. Also, show that A is the mid-point of the line segment BC.
39. Show that the area of a parallelogram having diagonals $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ is $5\sqrt{3}$ sq units.
40. If A(3, 5, -4), B(-1, 1, 2) andC(-5, -5, -2) are the vertices of a ΔABC , then find the direction cosines of AB, AC and BC.